The relation between

total radiation dose in gray [Gy] and

amount of X-photons/mm2

This is a first principle derivation of a rule of thumb.

Definition
The definition of the radiation dose unit “gray” [Gy] is

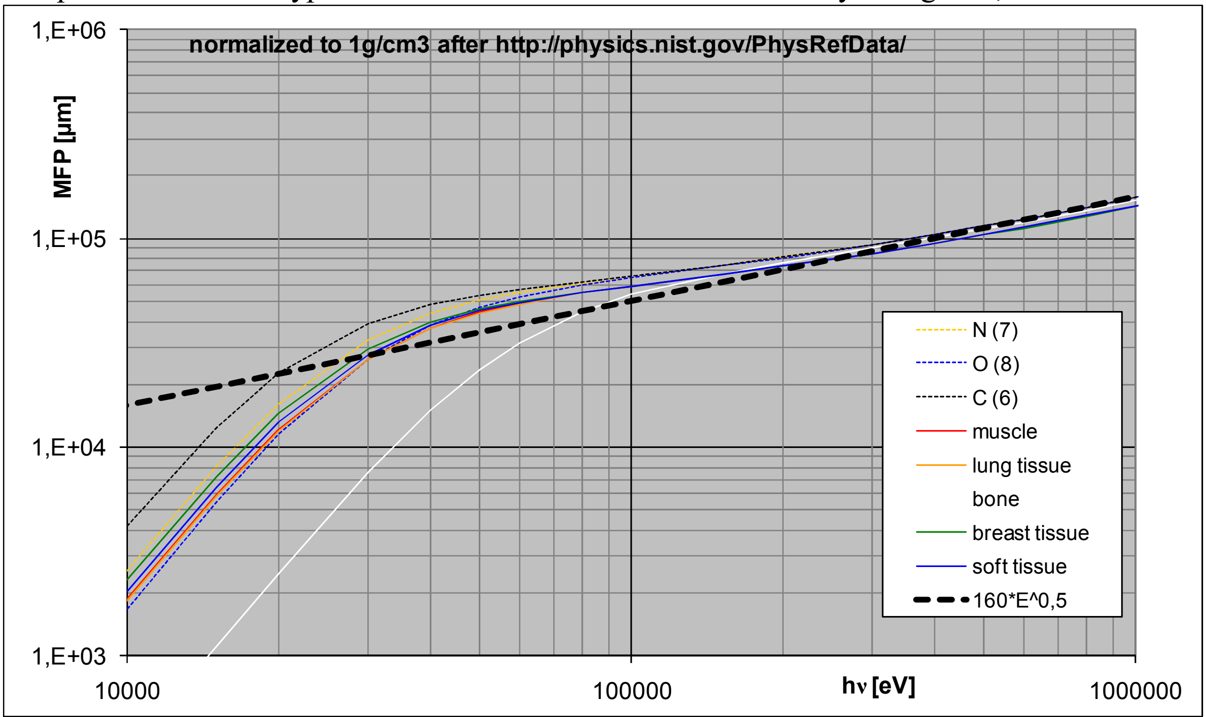
$$1 \text{ Gy} = 1 \frac{\text{J}}{\text{kg}} = 1 \text{ m}^2 \cdot \text{s}^{-2}$$

A dose of 1 Gy occurs when the radiation deposits 1 joule of energy in 1 kg of material.

Assumptions
We need following assumptions

- ⇒ This energy is sum of all absorbed photon energies hν [eV]
- ⇒ The material of interest here is “average human body tissue”, with a thickness t_{body}[cm]
- ⇒ we assume that the tissue has a mass density of 1 g/cm3
- ⇒ The kg of material is thought to be contained in the absorption length or “MFP” (mean free path) of the X-ray of that specific wavelength, if the MFP is smaller than the thickness, otherwise if is contained in the full thickness.

We make use of the observation that absorption of X-ray in tissues is more or less independent of tissue type. When we normalize to a mass density of 1 g/cm^3, one finds:



The fit

$$\text{MFP}\ [\mu\text{m}] = 160 * (h\nu\ [\text{eV}])^{0.5}$$

is a useful approximation in the range 10 to 1000 keV, the range of typical medical X-ray.

Example
A 400keV photon has a MFP of 10cm.
Hence, if the radiation is smeared over 100cm^2, it is completely absorbed in 1dm^3, or 1kg of mass, assuming the body is much thicker than this 10cm.

thus

One 400keV photon = an energy of 1.6E-19*4E5 J
One 400keV photon/100cm^2 absorbed over 10cm = a dose of 1.6E-19*4E5 Gy

thus

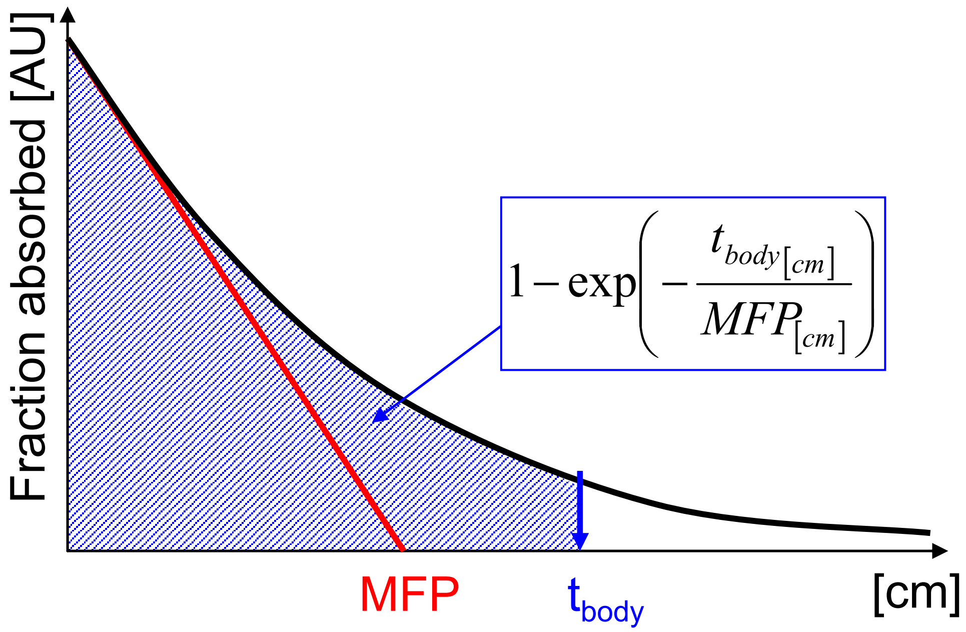
a 400keV photon, in a very (infinitely) thick average human tissue yields:
1 photon/mm2 = 6.4E-10 Gy

or

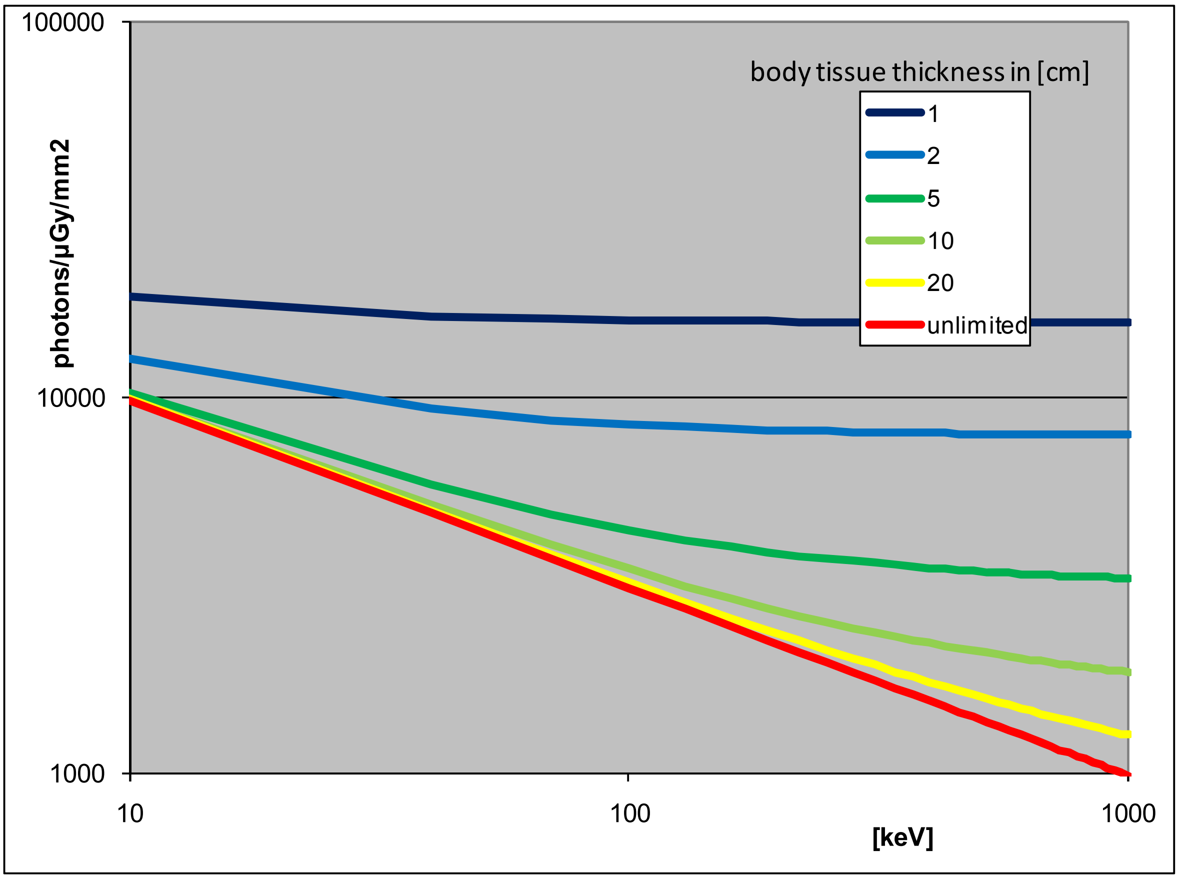
1560 photons/mm2/µGy

If the tissue is not so thick, the absorption is incomplete. For the same dose, one needs more photons. The above number must be divided by the fraction absorbed, which is

$$fraction\_absorbed = 1 - \exp\left(-\frac{t_{body[cm]}}{MFP_{[cm]}}\right)$$



This is generalized to different energies and tissue thicknesses in the following graph



Approximate formula
The formula behind this graph is

$$P\ [\text{Photons}/\mu\text{Gy}/\text{mm}^2] = \frac{P_{INF}}{1 - \exp\left(-\frac{t_{body\,[cm]}}{MFP_{[cm]}}\right)}$$

Where P_{INF} is the value for an infinitely thick (sufficiently thick to have full absorption) tissue.

$$P_{INF}\ [\text{Photons}/\mu\text{Gy}/\text{mm}^2] = 31200 / \sqrt{h\nu[keV]}$$

And

$$MFP_{[cm]} = 160 \cdot \sqrt{h\nu[eV]}$$

Which can be simplified to a rule of thumb

Rule of thumb

⇒ When the t_{body} >> MFP
P [Photons/µGy/mm^2] = 31200 / SQRT(hν[keV])

⇒ When the t_{body} << MFP
P [Photons/µGy/mm^2] = 158000 / t_{body}[cm])

Notes

⇒ Thicknesses used here assume "mass thickness", being equal to geometrical thickness if density is 1g/cm3. For a different density adjust the thickness accordingly.

⇒ The photons quoted are hitting the body/tissue. When the body is not present, the same amount hits the detector. In a magnifying configuration, the flux must be scaled accordingly.

⇒ No obstruction between body tissue and detector. Any extra obstruction (grid, …) will decrease the photons per gray as seen by the detector.

⇒ The formula is reasonably accurate. The greatest error will be made for low energies (<10keV) AND thin bodies.